

Physics 127a: Class Notes

Lecture 11: Entropy, Information and Maxwell's Demon

Gibbs Entropy

For the canonical ensemble using

$$P_n = Q_N^{-1} e^{-\beta E_n} \quad (1a)$$

$$A = -kT \ln Q_N = U - TS \quad (1b)$$

it is easy to show

$$S = -k \sum_n P_n \ln P_n \quad (2)$$

an expression directly relating the thermodynamic entropy to the microscopic probability distribution. The same result applies in the grand canonical ensemble, and the expression is consistent with the original definition in the microcanonical ensemble ($P_n = 1/\Omega$ over Ω accessible states).

This is known as the *Gibbs expression* for the entropy. Note that it presents the entropy from a slightly different perspective—the entropy of the *ensemble* representing a physical situation rather than the entropy of a *system*. Those of you who have encountered Shannon information theory will find Eq. (2) familiar: you can think of the entropy as the information to be learned on a thermodynamic system by determining which microstate the system is actually in. (Note, of course, the concept of entropy preceded information theory.)

You might wonder, since Eq. (2) only depends on the probabilities within the ensemble, whether we can use this as a microscopic definition of the entropy from which we can simply and unambiguously derive the increase of entropy directly from the laws of mechanics. The answer is no.

Consider a classical continuum system where \sum_n is replaced by the integral over phase space. Dropping constants we then have

$$S \propto -k \int \rho \ln \rho \quad (3)$$

with ρ the phase space distribution and the integral is over $6N$ dimensional phase space. Does this entropy increase in the dynamics? The dynamics of ρ under the Hamiltonian of the system satisfies Liouville's theorem

$$\frac{d\rho}{dt} = 0. \quad (4)$$

Consequently the Gibbs entropy is *constant* in the dynamics $dS/dt = 0$! The phase space density is redistributed, but its value (moving with the phase space coordinate) does not change, and so the integral is unchanged. To retrieve the increase of entropy we have to introduce some coarse graining or statistical ideas on top of the dynamics. This is not surprising: the microscopic dynamics is invariant under time reversal, whereas the increase of entropy is not. We can certainly use the Gibbs expression to calculate the entropy for the equilibrium ρ , but the use of the expression as a microscopic one for distributions P_n far away from equilibrium ones is a much more uncertain question.

Maxwell's Demon

For those of you who *want* to read more about Maxwell's demon I've listed some references below:

- “*The Mathematical Theory of Communication*” by C.E. Shannon (call no. TK5101 .S45 1949 — Millikan Reserve): introduction to information theory.
- R. Landauer, Nature **335** 779 (1988): where he reviews the idea of dissipation associated with memory erasure.
- C.H. Bennett, IBM J. Res. Dev. **32** 16 (1988), “Notes on the history of reversible computation”
- W.H. Zurek, Phys. Rev. A**40** 4731 (1989): discusses algorithmic randomness.
- Feynman’s “*Lectures on Physics*”, Vol. 1, Ch.46: the thermal ratchet.
- “*Maxwell's Demon*” by H.S. Leff and A.F. Rex: a good introduction and compilation of many of the important papers (from 1874 on!). The first chapter is a nice historical review, and the book includes reprints of the papers listed above, and useful additional ones by Landauer. Chapter 4 contains a number of interesting papers by Bennett on the thermodynamics of computing. This book isn’t on reserve, but you can borrow my copy (terms: must be returned to the bookshelf in Sloan Annex at noon and 4 pm. everyday).