Physics 161: Homework 3

(19 January 2000, due 26 January)

Problems

1. Consider the power spectrum for the van der Pohl oscillator driven at frequency 1.15 available under the "Homework" link on the website. We might be interested in the question of whether the motion is periodic, i.e. locked to the drive frequency, or quasiperiodic, i.e. the internal oscillations at a frequency $\omega_{in} \simeq 1$ and the drive at a frequency $\omega_D = 1.15$ both present (together with other frequencies, $m\omega_{in} \pm n\omega_D$ with m, n integers, given by the non-linear interaction).

   (a) With the values of *dt*, *Interval*, and *Points* initially set for the applet would you expect to be able to resolve this question and why? (You might want to read the instructions and discussion of the diagnostics for the *Odes* applet on the website.)

   (b) Change the values of any of *dt*, *Interval*, *Points*, and *Window* appropriately to get the best information you can on the spectrum, and make a sketch of what you see.

   (c) Is the motion probably periodic or quasiperiodic?

2. The Hénon map is given by:

$$\begin{aligned} x_{n+1} &= 1 - ax_n^2 + y_n \\ y_{n+1} &= bx_n \end{aligned}$$

   Common parameters used are $a = 1.4$ and $b = 0.3$.

   (a) Set up a scheme for calculating the two Lyapunov eigenvalues of the map, i.e. do all the algebra to make explicit what you would iterate and how you would calculate the exponents.

   (b) Use the *2dmap* demonstration to study the variation of the largest Lyapunov exponent on the Hénon attractor with a between 0.2 and 1.4 for $b = 0.3$ and relate the variation to the nature of the attractor. (Note: make sure you hit the *Reset* button on the applet after you change a to restart the calculation of the Lyapunov exponent. For the periodic orbits with a few points only you might want to increase *Mark* to 0.5 to make the points more obvious.)

3. Consider the two dimensional map (introduced by Kaplan and Yorke)

$$x_{n+1} = ax_n \bmod 1$$

$$y_{n+1} = by_n + \cos(2\pi x_n)$$

   for $a = 3$.

   (a) Show that the map has an attractor which lies in $|y| \le 1/(1 - |b|)$ provided $|b| < 1$.

   (b) What is the Jacobean of the map at the point (x, y)? What are the local directions of expansion and contraction?

   (c) What are the Lyapunov exponents of the map?

   (d) Now iterate the map numerically for $b = \frac{1}{4}$ and for $b = \frac{1}{2}$ e.g. using the program *2dmap* on the website (this map is labelled the "Yorke Map" there). Do the nuerical values of the Lyapunov exponents agree with what you calculated?

   (e) Sketch the attractor of the map obtained numerically for $b = \frac{1}{4}$ and for $b = \frac{1}{2}$. Do you see any qualitative difference between the attractors at these two values?